

Example of the Fourth - Order Taylor Method

IVP : $\frac{dy}{dx} = 2xy$

$y(0) = 2$

$y'' = 2xy' + 2y$

$y''' = 2xy'' + 4y'$

$y^{(4)} = 2xy''' + 6y''$

Step size : $h = 0.1$

$\frac{1}{2} h^2 = 0.005$

$\frac{1}{6} h^3 = 0.001/6$

$\frac{1}{24} h^4 = 0.0001/24$

x_0	0	0.1	0.2	0.3	0.4
y_0	2	2.020100	2.081619	2.188341	2.347006
y'	0	0.404020	0.832648	1.313005	1.877605
y''	4	4.121004	4.496297	5.164485	6.196095
$y^{(3)}$	0	2.441281	5.129109	8.350709	12.467294
$y^{(4)}$	24	25.214080	29.029426	35.997335	47.150406
hy'	0	0.040402	0.083265	0.131300	0.187760
$\frac{1}{2} h^2 y''$	0.02	0.020605	0.022481	0.025822	0.030980
$\frac{1}{6} h^3 y^{(3)}$	0	0.000407	0.000855	0.001392	0.002078
$\frac{1}{24} h^4 y^{(4)}$	0.0001	0.000105	0.000121	0.000150	0.000196
Δy	0.0201	0.061519	0.106722	0.158665	0.221015

x_0	0.5	0.6	0.7	0.8	0.9
y_0	2.568021	2.866607	3.264545	3.792818	4.495582
y'	2.568021	3.439928	4.570364	6.068509	8.092048
y''	7.704063	9.861128	12.927600	17.295252	23.556850
$y^{(3)}$	17.976147	25.593067	36.380094	51.946441	74.770520
$y^{(4)}$	64.200525	89.878447	128.497730	186.885817	275.928036
hy'	0.256802	0.343993	0.457036	0.606851	0.809205
$\frac{1}{2} h^2 y''$	0.038520	0.049306	0.064638	0.086476	0.117784
$\frac{1}{6} h^3 y^{(3)}$	0.002996	0.004266	0.006063	0.008658	0.012462
$\frac{1}{24} h^4 y^{(4)}$	0.000268	0.000374	0.000535	0.000779	0.001150
Δy	0.298586	0.397938	0.528273	0.702764	0.940600

x	1
y	5.436183

Comparison of the results obtained from the Fourth -
 - Order Taylor Method by using various step sizes.

IVP ; $y' = 2xy$, $y(0) = 2$

	$n=1$ $h=0.1$	$n=2$ $h=0.05$	$n=5$ $h=0.02$	$n=10$ $h=0.01$	Correct y^*
x	y	y	y	y	
0	2	2	2	2	2
0.1	2.020100	2.020100	2.020100	2.020100	2.020100
0.2	2.081619	2.081621	2.081622	2.081622	2.081622
0.3	2.188341	2.188348	2.188349	2.188349	2.188349
0.4	2.347006	2.347021	2.347022	2.347022	2.347022
0.5	2.568021	2.568049	2.568051	2.568051	2.568051
0.6	2.866607	2.866655	2.866659	2.866659	2.866659
0.7	3.264545	3.264626	3.264632	3.264632	3.264632
0.8	3.792818	3.792952	3.792961	3.792962	3.792962
0.9	4.495582	4.495800	4.495816	4.495816	4.496816
1	5.436183	5.436537	5.436563	5.436564	5.436564

* $y = 2e^{x^2}$